Quantum algorithm for the Laughlin wave function

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Outline

1. Introduction and motivation
2. Quantum circuit for the m=1 Laughlin state
3. Properties of our algorithm
4. Experimental realization
5. Conclusions

Introduction

Quantum and classical computations

Quantum computer

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exponentialy faster

Classical computer

Shor's algorithm (factorization), Grover's algorithm (search), ...

Problem: these algorithms require thousands of perfectly controlled qubits.

Simulation of QM by means of a QC

Another application of the QC would be the simulation of quantum systems.

Quantum Computer

with only tens of controlled qubits

→

Simulation

of strongly correlated systems of few particles

(FQHE, spin models, etc.)

To do this, new quantum algorithms are required.

Simulation of QM

DISENTANGLERS

Verstraete, Cirac and Latorre (arXiv:0804.188809, PRA 79, 032316 2009)

$$\mathcal{H} = \mathcal{U}_{dis}\, \tilde{\mathcal{H}}\, \mathcal{U}_{dis}^{\dagger}$$

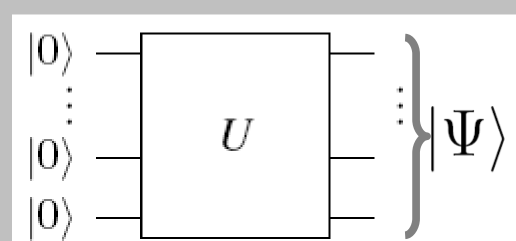
$$\tilde{\mathcal{H}} = \sum_i \omega_i \sigma_i^z$$

Excited eigenstates, dynamics and finite temperature.

$$e^{-it\mathcal{H}} = \mathcal{U}_{dis} e^{-it\tilde{\mathcal{H}}} \mathcal{U}_{dis}^{\dagger}$$

STATE PREPARATION

Preparation of a state independently of the dynamics.

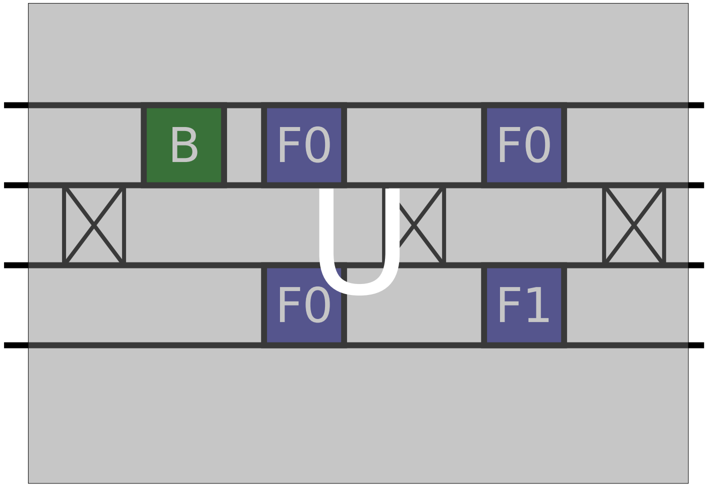


$$|\Psi\rangle = U\,|\Psi_0\rangle$$

Measure of correlations, understand the physics behind it.

Requirements of the circuit

- Single and two qudit gates (each qudit gate can be decomposed in qubits).
- Local gates.
- Polynomial number of gates.



Quantum circuit for the $m=1$ Laughlin state

The Laughlin state

$$\Psi_L^{(m)}(z_1, \ldots, z_n) \sim \prod_{i<j}(z_i - z_j)^m e^{-\sum_{i=1}^N |z_i|^2/2}$$

- It was postulated by Laughlin as the ground state of fractional quantum Hall effect (FQHE).
- It is a strongly correlated state (for m>1).
- It exhibits a considerable von Neumann entropy between any of its possible partitions.

The m=1 Laughlin state

In particular, the m=1 case (first quantization)

$$\Psi_L(z_1,\ldots,z_n) = \frac{1}{\sqrt{n!}} \sum_{\mathcal{P}} \mathrm{sign}(\mathcal{P}) \varphi_{a_1}(z_1)\ldots\varphi_{a_n}(z_n)$$

where $\varphi_l(z) = \langle z|l\rangle = z^l \exp(-|z|^2/2)/\sqrt{\pi l!}$ are the Fock-Darwin angular momentum eigenstates.

Notice that it is **not** a strongly correlated system, but a set of non-interacting fermions.

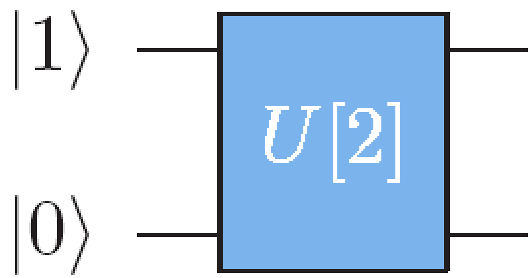
$$|\Psi_L\rangle = |1\,1\,\ldots 1\rangle = a_0^\dagger a_1^\dagger \ldots a_{n-1}^\dagger |0\rangle$$

where $\{a_i, a_j^\dagger\} = \delta_{ij}$ and $a_l^\dagger |0\rangle = |l\rangle$.

n=2 case

$$\Psi_L^{(2)}(z_1, z_2) = \frac{1}{\sqrt{2}}\left(\varphi_1(z_1)\varphi_0(z_2) - \varphi_0(z_1)\varphi_1(z_2)\right)$$

$$|\Psi_0^{(2)}\rangle = |0, 1\rangle$$



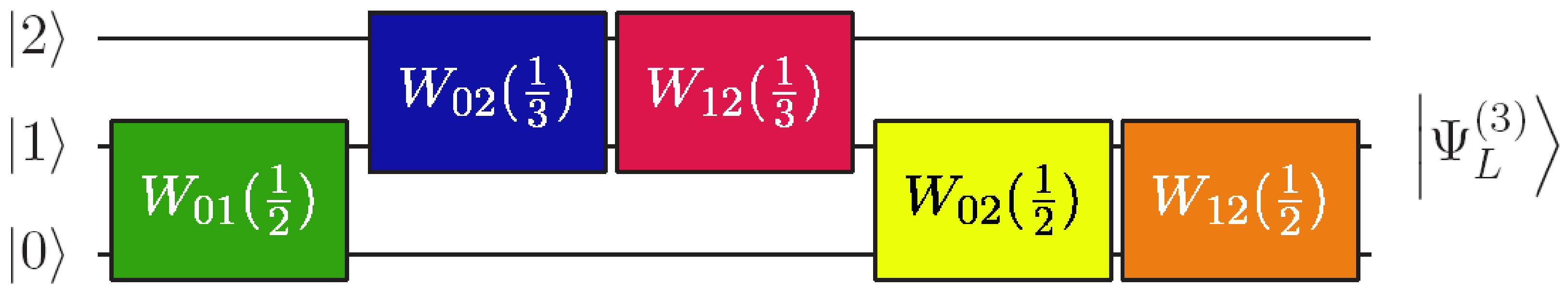
$$\left|\Psi_L^{(2)}\right\rangle = \frac{1}{\sqrt{2}}\left(|0, 1\rangle - |1, 0\rangle\right)$$

$$U[2] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\{|0, 0\rangle, |0, 1\rangle, |1, 0\rangle, |1, 1\rangle\}$$

n=3 case

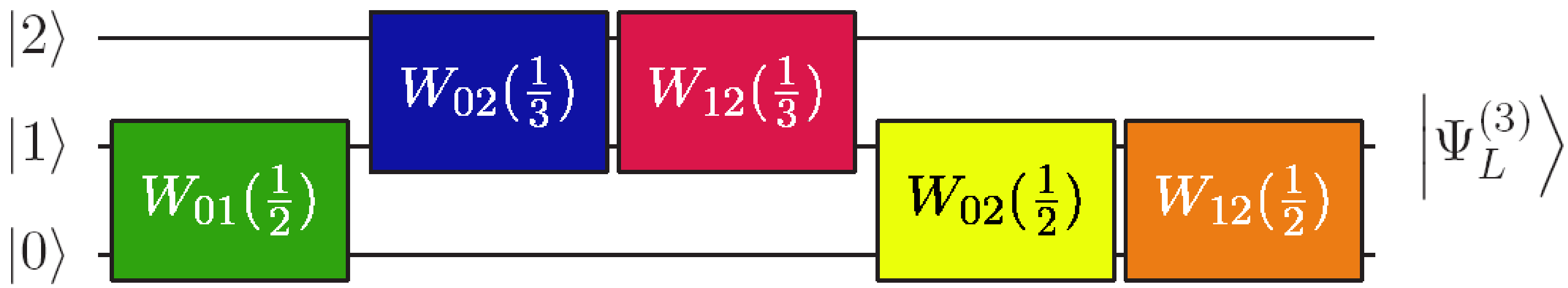
$$\Psi_L^{(3)}(z_1, z_2, z_3) = \frac{1}{\sqrt{6}} \begin{vmatrix} \varphi_0(z_1) & \varphi_0(z_2) & \varphi_0(z_3) \\ \varphi_1(z_1) & \varphi_1(z_2) & \varphi_1(z_3) \\ \varphi_2(z_1) & \varphi_2(z_2) & \varphi_2(z_3) \end{vmatrix}$$



The definition of *W*-gates

$$W_{ij}(p)|ij\rangle = \sqrt{p}|ij\rangle - \sqrt{1-p}|ji\rangle$$
$$W_{ij}(p)|ji\rangle = \sqrt{p}|ji\rangle + \sqrt{1-p}|ij\rangle \qquad \text{for } i < j$$

$$W_{ij}|kl\rangle = |kl\rangle \text{ if } (k,l) \neq (i,j)$$

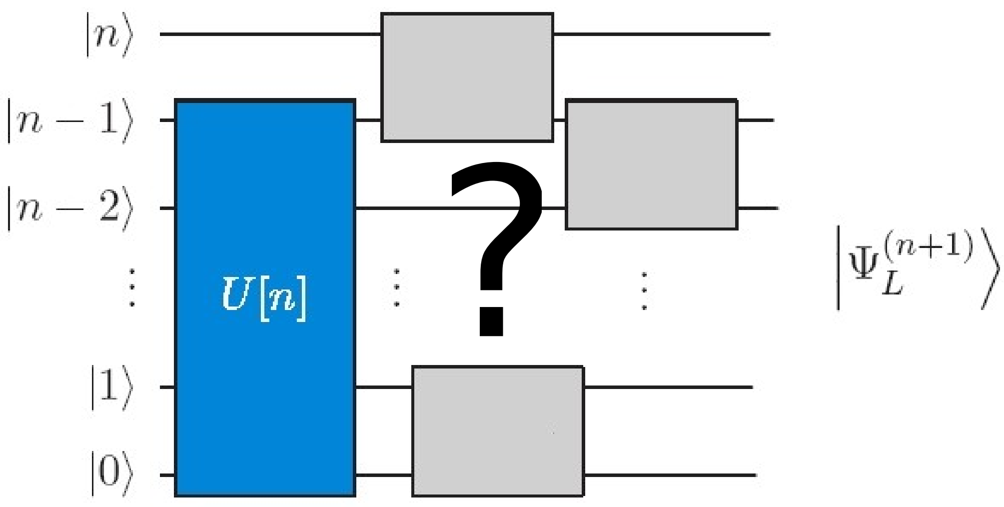


Evolution of the initial state step by step,

$$\left|\begin{matrix}2\\1\\0\end{matrix}\right\rangle \rightarrow \frac{1}{\sqrt{2}}\left|\begin{matrix}2\\1\\0\end{matrix}\right\rangle - \frac{1}{\sqrt{2}}\left|\begin{matrix}2\\0\\1\end{matrix}\right\rangle \rightarrow \frac{1}{\sqrt{6}}\left|\begin{matrix}2\\1\\0\end{matrix}\right\rangle - \frac{1}{\sqrt{3}}\left|\begin{matrix}1\\2\\0\end{matrix}\right\rangle - \frac{1}{\sqrt{6}}\left|\begin{matrix}2\\0\\1\end{matrix}\right\rangle + \frac{1}{\sqrt{3}}\left|\begin{matrix}0\\2\\1\end{matrix}\right\rangle$$

$$\rightarrow \frac{1}{\sqrt{6}}\left|\begin{matrix}2\\1\\0\end{matrix}\right\rangle - \frac{1}{\sqrt{6}}\left|\begin{matrix}2\\0\\1\end{matrix}\right\rangle - \frac{1}{\sqrt{6}}\left|\begin{matrix}1\\2\\0\end{matrix}\right\rangle + \frac{1}{\sqrt{6}}\left|\begin{matrix}1\\0\\2\end{matrix}\right\rangle + \frac{1}{\sqrt{6}}\left|\begin{matrix}0\\2\\1\end{matrix}\right\rangle - \frac{1}{\sqrt{6}}\left|\begin{matrix}0\\1\\2\end{matrix}\right\rangle$$

$$\boxed{\frac{1}{\sqrt{6}}\left(\left|\begin{matrix}2\\1\\0\end{matrix}\right\rangle - \left|\begin{matrix}2\\0\\1\end{matrix}\right\rangle - \left|\begin{matrix}1\\2\\0\end{matrix}\right\rangle + \left|\begin{matrix}1\\0\\2\end{matrix}\right\rangle + \left|\begin{matrix}0\\2\\1\end{matrix}\right\rangle - \left|\begin{matrix}0\\1\\2\end{matrix}\right\rangle\right)}$$



U(n) generates a superposition of the *n*! permutations

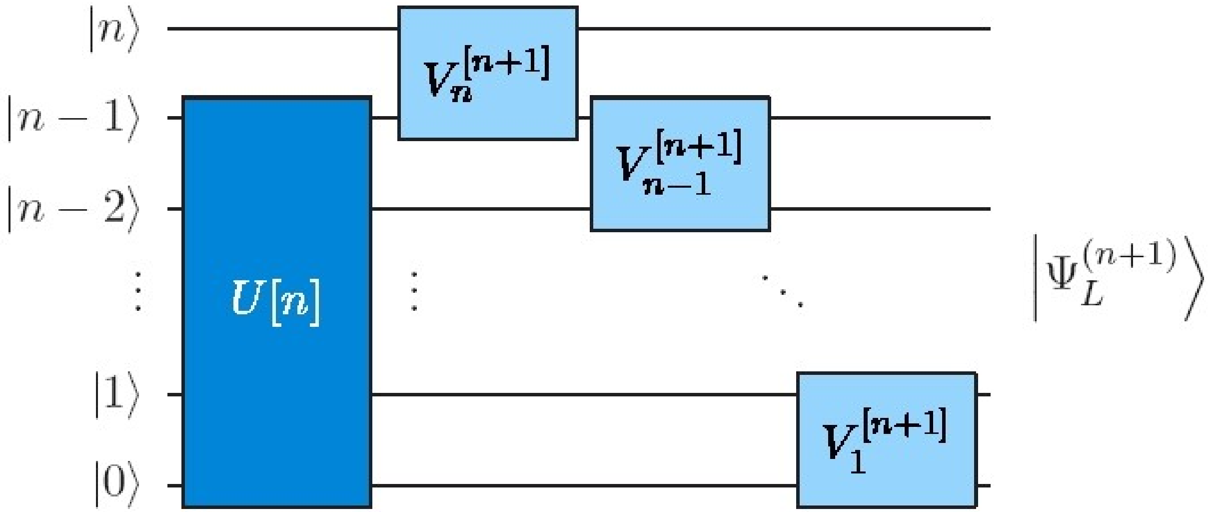
$$\left.\begin{array}{ccccc} 0 & 1 & 2 & \dots & (n-1) \\ 1 & 0 & 2 & \dots & (n-1) \\ \vdots & & & & \vdots \\ (n-1) & (n-2) & \dots & 1 & 0 \end{array}\right\} n!$$

We can achieve the set of permutations of *n+1* elements performing the following series of simple transpositions

$$\left.\begin{array}{ccccc} a_1 & a_2 & \dots & a_n & n \\ a_1 & a_2 & \dots & n & a_n \\ & \vdots & & \vdots & \\ n & a_1 & a_2 & \dots & a_n \end{array}\right\} (n+1)\, n! = (n+1)!$$

Arbitrary *n*

This suggests the following structure...



with

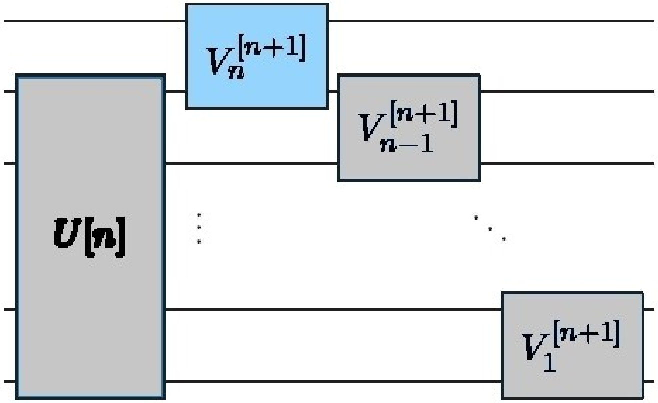
$$V_n^{[n+1]} = W_{0n}(p)W_{1n}(p)\dots W_{n-1\,n}(p)$$

$$V_k^{[n+1]} = \prod_{i=0}^{n-1} W_{in}$$

Arbitrary *n*

Wich is the weight of the *V*-gates?

$$\frac{1}{\sqrt{n!}}\begin{pmatrix} n \\ a_n \\ a_{n-1} \\ \vdots \\ a_1 \end{pmatrix} \longrightarrow \sqrt{\frac{p}{n!}}\begin{pmatrix} n \\ a_n \\ a_{n-1} \\ \vdots \\ a_1 \end{pmatrix} - \sqrt{\frac{1-p}{n!}}\begin{pmatrix} a_n \\ n \\ a_{n-1} \\ \vdots \\ a_1 \end{pmatrix}$$



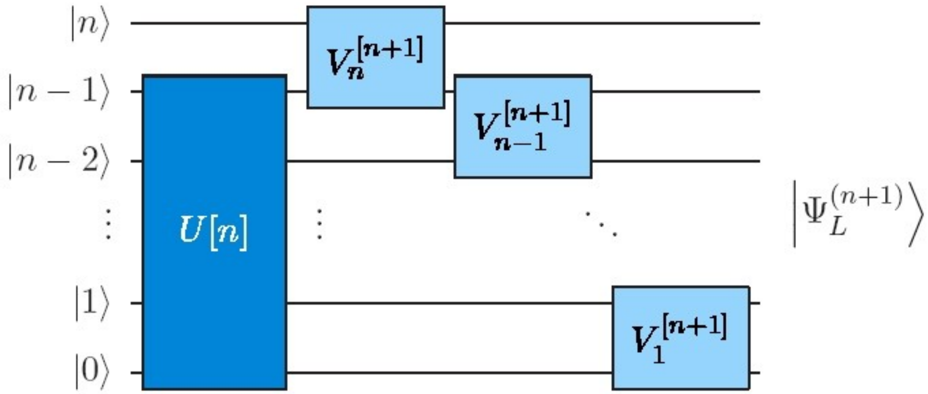
$$\sqrt{\frac{p}{n!}} = \frac{1}{\sqrt{(n+1)!}} \quad \Rightarrow \quad p = \frac{1}{n+1}$$

We can follow the same argument for the rest of gates

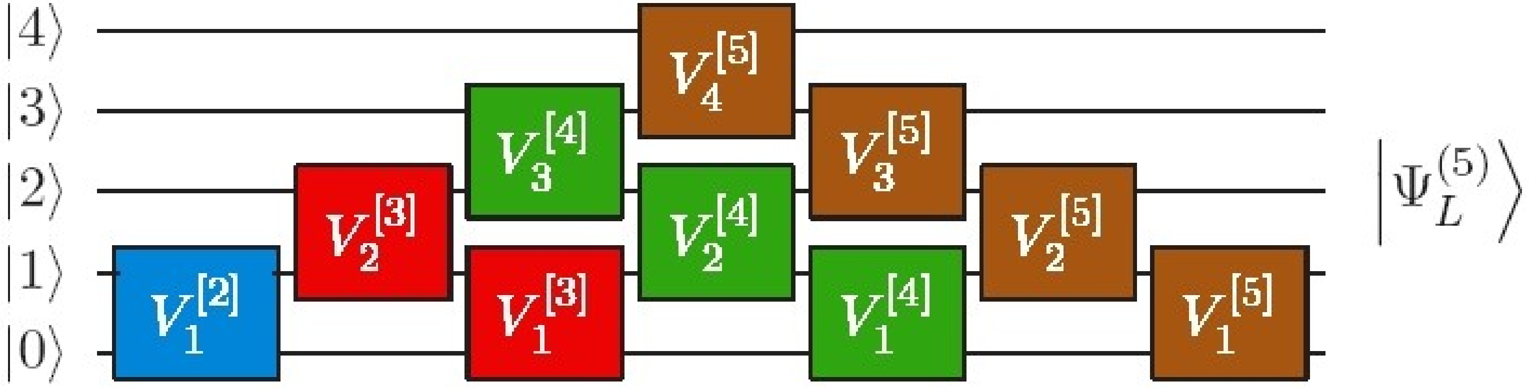
$$p\left(V_k^{[n+1]}\right) = \frac{1}{k+1}$$

Arbitrary *n*

This recursive structure



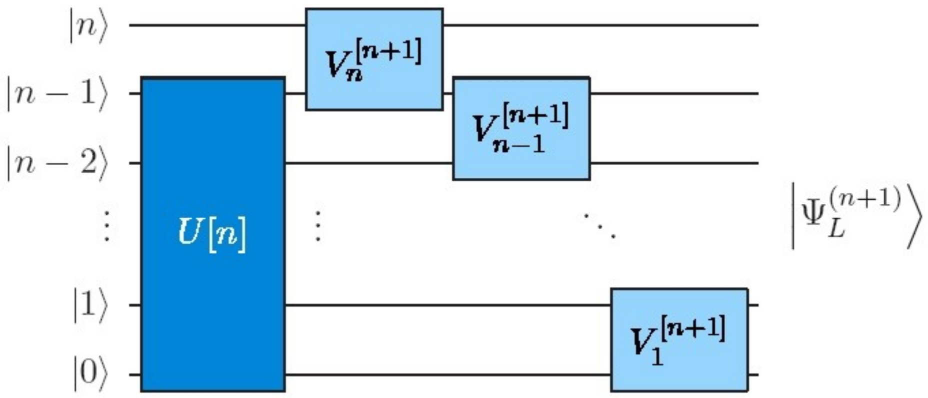
can be rewritten as



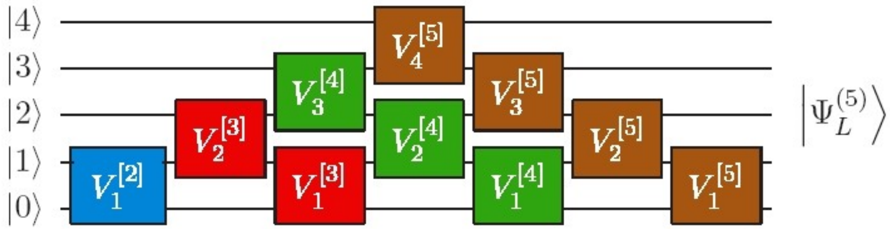
Properties of our algorithm

Number of gates and depth

The number of gates *N(n)* and the depth *D(n)* of the circuit scale with *n* as



$$N(n) = (n-1) + N(n-1) = \frac{n(n-1)}{2}$$



$$D(n) = D(n-1) + 2 = 2n - 3,$$

Optimal circuit

The number of gates $N(n)=n(n-1)/2$ of our proposal is optimal.

0,1,2,...,(n-2),(n-1)

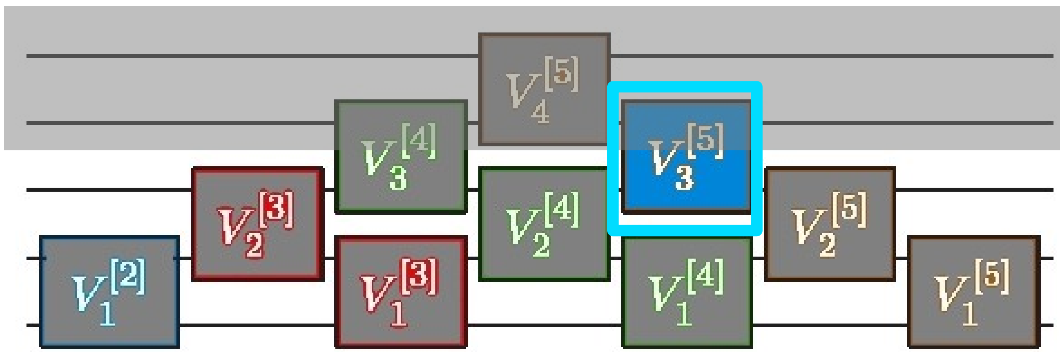
↓

(n-1),(n-2),...,,2,1,0

The number of simple transpositions required to connect both sequences is $n(n-1)/2$.

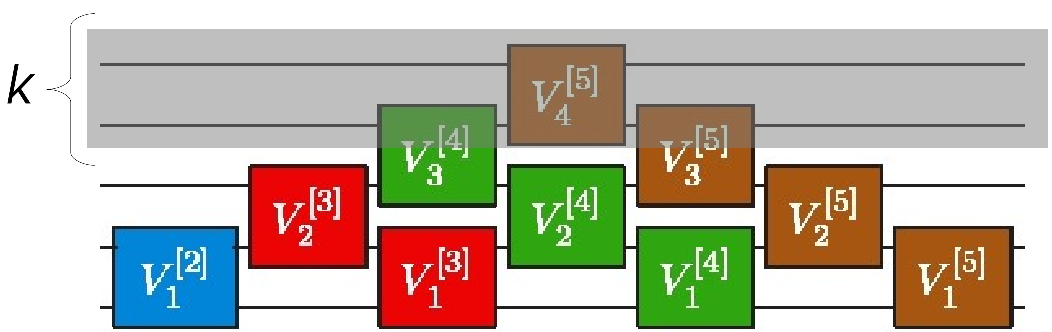
Entanglement entropy

How much entanglement does a *V*-gate generate?



$$\Delta S\left(V_k^{[n]}\right) = \log_2\left(\frac{n}{n-k}\right)$$

The Von Neumann entropy between *k* particles and the rest of the system is



$$S_{n,k} = \sum_{n'=k+1}^{n} \Delta S\left(V_k^{[n']}\right) = \log_2\binom{n}{k}$$

Result already found by Iblisdir, Latorre and Orus (PRL 98:060402, 2007)

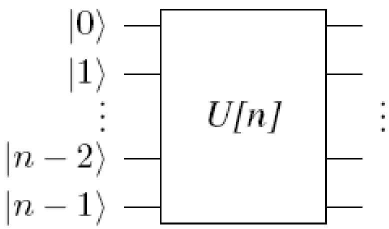
Symmetrization

We can transform our antisymmetrization circuit into a symmetrization one.

$$\sqrt{p}|ij\rangle \mp \sqrt{1-p}|ji\rangle \quad \rightarrow \quad \sqrt{p}|ij\rangle \pm \sqrt{1-p}|ji\rangle$$

Another possibility would be

i) to invert the order of the input state



ii) Apply the gates $W_{n-1-i,n-1-j}$ instead of W_{ij}

Experimental realization

Experimental realization

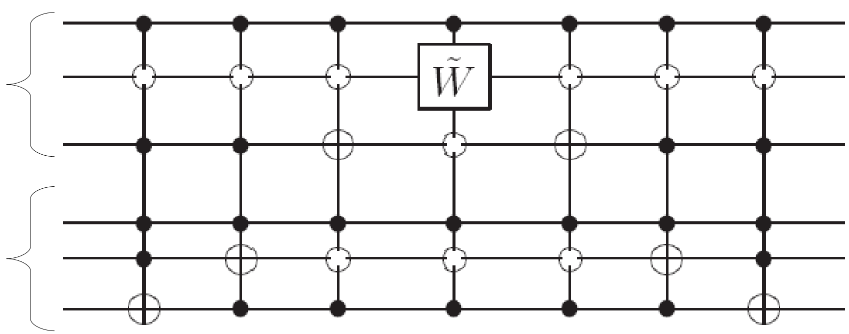
Our *W*-gates, that act on qudits, can be decomposed on multiqubit gates acting on qubits.

i) Encoding of qudits in terms of qubits.

$$|i\rangle = |i_r\rangle \dots |i_2\rangle |i_1\rangle \,,$$

where $i = \sum_{k=1}^{r} 2^{k-1}$.

ii) Example of the decompositon of a *W*-gate.



Any of these multiqubit gates can be implemented by means of O(r) of two-qubit gates.

Conclusions

Conclusions

1) We have presented a quantum circuit that generates the m=1 Laughlin wave function for an arbitrary number of qudits.

2) We have shown that our proposal is optimal and can be realized with a number of $O(n^3 (\log n)^2)$ C-NOT and single qubit gates.

3) Although we have some particular results for $m>1$, a circuit for an arbitrary n is still missing.

Thank you very much for your attention