# Symmetry breaking in small rotating cloud of trapped ultracold Bose atoms

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## Abstract

We study the signatures of rotational and phase symmetry breaking in small rotating clouds of trapped ultracold Bose atoms by looking at rigorously defined condensate wave function. Rotational symmetry breaking occurs in narrow frequency windows, where the ground state of the system has degenerated with respect to the total angular momentum, and it leads to a complex wave function that exhibits vortices clearly seen as holes in the density, as well as characteristic vorticity. Phase symmetry (or gauge symmetry) breaking, on the other hand, is clearly manifested in the interference of two independent rotating clouds.

## Introduction

### Symmetries in BEC's

For Bose-Einstein condensates (BEC) two symmetries play a particular role:

- $U(1)$ phase symmetry, which is broken when the system has to choose a phase.
- $SU(2)$ (or $SO(3)$) rotational symmetry: it is broken when the $|GS\rangle$ has a structure that is not rotational invariant as the Hamiltonian.

In the large $N$ limit, one breaks these symmetries by hand, since we use a classical field, also called order parameter, that characterizes the system in a proper way [1] (it is the solution of the **Gross Pitaevskii (GP) equation**). It has an arbitrary, but fixed phase (phase symmetry breaking), and for rotating systems with more than one vortex it exhibits a vortex array (rotational symmetry breaking).

### Measurement in BEC's

It is not clear what does a measurement mean in a single shot:

- the density,

or,

- the n-correlation function.

The modern point of view [2] implies that two BEC's with fixed $N$ each one, will produce a well defined interference pattern of fringes as a result of the measurement in only one shot (comparable with the calculated n-correlation function) in contrast with the density, which would be obtained as a mean image of several shots. This is a phase symmetry breaking effect.

If this is the case, a system with well defined angular momentum $L$, such that presents different density and n-correlation functions, could be used to determine the meaning of a measurement in only one shot (breaking of rotational symmetry).

The problem is that for large N-systems, the total angular momentum of the stationary states is not well defined. This is a motivation to study systems with few particles, in which is possible to have a well defined angular momentum, and in this way to obtain a test of the meaning assigned to the measurement.

### Interpretation of interferences

The previous one is the interpretation for the interference pattern formation suggested by Mullin and collaborators. There is another possibility discussed by Cederbaum et al. [3], according to which interference pattern appears if one includes interaction during the time-of-flight even for states that initially are Fock states. We will present in this work an alternative interpretation.

## Systems with few particles

### Description of our system

We consider a two-dimensional system of few Bose atoms trapped in a parabolic rotating trap around the z-axis. The rotating frequency $\Omega$ is strong enough to consider the Lowest Landau Level regime with atoms interacting via contact forces. Thus, the Hamiltonian in the rotating reference frame reads [4],

$$H = \sum_{i=1}^{N} \left[ \frac{(\vec{p} - \frac{e}{c}\vec{A}^*)_i^2}{2M} + \frac{1}{2} M(\omega_\perp^2 - \Omega^2) r_i^2 \right] + g \sum_{i<j} \delta(\vec{r}_i - \vec{r}_j), \quad (1)$$

where $\vec{r} = (x, y)$, $\omega_\perp$ is the trap frequency, $\vec{A}^* = \frac{M\Omega c}{e} \hat{z} \times \vec{r}$ is the vector potential, $\hat{z}$ is the unitary vector along the $Z$ direction and $\vec{B}^* = \vec{\nabla} \times \vec{A}^* = \frac{2M\Omega c}{e}\hat{z}$ is the effective magnetic field of an equivalent system of electrons submitted to a magnetic field perpendicular to the $XY$ plane (we use here the symmetric gauge).
We choose the appropriate Fock–Darwin single particle (sp) wave functions as the basis in order to represent all operators [5],

$$\varphi_l(\vec{r}) = e^{il\theta} r^l e^{-r^2/2} / \sqrt{\pi l!} \quad (2)$$

where $l$ labels the single particle angular momentum; length unit is here $\lambda = \sqrt{\hbar/(m\omega_\perp)}$.

### Order parameter

Our goal is **to obtain in a precise way a complex scalar field that models efficiently the system**, and allows to reproduce the important features, such as the vortex states. Our analysis is performed using the **exact diagonalization** formalism, valid for arbitrary interactions and densities.
With this method we find the ground state as a multi-particle WF's and loose the intuitive picture provided by the mean field order parameter.
The way to know if there is a "macro-occupied" SP wave function in the ground state $|GS\rangle$ is to look at the eigenvalues and eigenvectors of the one body density matrix (OBDM) $n^{(1)}(\vec{r}, \vec{r'}) = \langle GS \mid \hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r'})|GS\rangle$,

$$\int d\vec{r'} n^{(1)}(\vec{r}, \vec{r'}) \psi_l^*(\vec{r'}) = n_l \psi_l^*(\vec{r}), \quad (3)$$

with $\hat{\Psi}$ being the field operator. If there exists a relevant eigenvalue $n_1 \gg n_l$ for $l = 2, 3, \ldots$, then

$$\sqrt{n_1} \psi_1(\vec{r}) e^{i\phi_1} \quad (4)$$

plays the role of the order parameter of the system, where $\phi_1$ is an arbitrary constant phase. The WF may be expanded in the form $\psi_1(\vec{r}) = \sum_{l=0}^{m} \beta_{1l} \varphi_l(\vec{r})$,

## Results

### Characterization of vortices

At certain values of $\Omega$ degeneracy takes place and vortex states without circular symmetry (except the case of only one centered vortex) are possible [6]. Then, the order parameter $\psi_1$ provides a non-ambiguous way to characterize vortices, since:

- it shows dimples in the density profile,
- the local phase $S(\vec{r})$ (defined as $\psi_1(\vec{r}) = \mid \psi_1(\vec{r}) \mid e^{iS(\vec{r})}$) indicates the position of each vortex by the change on multiples of $2\pi$ when moving around each one.

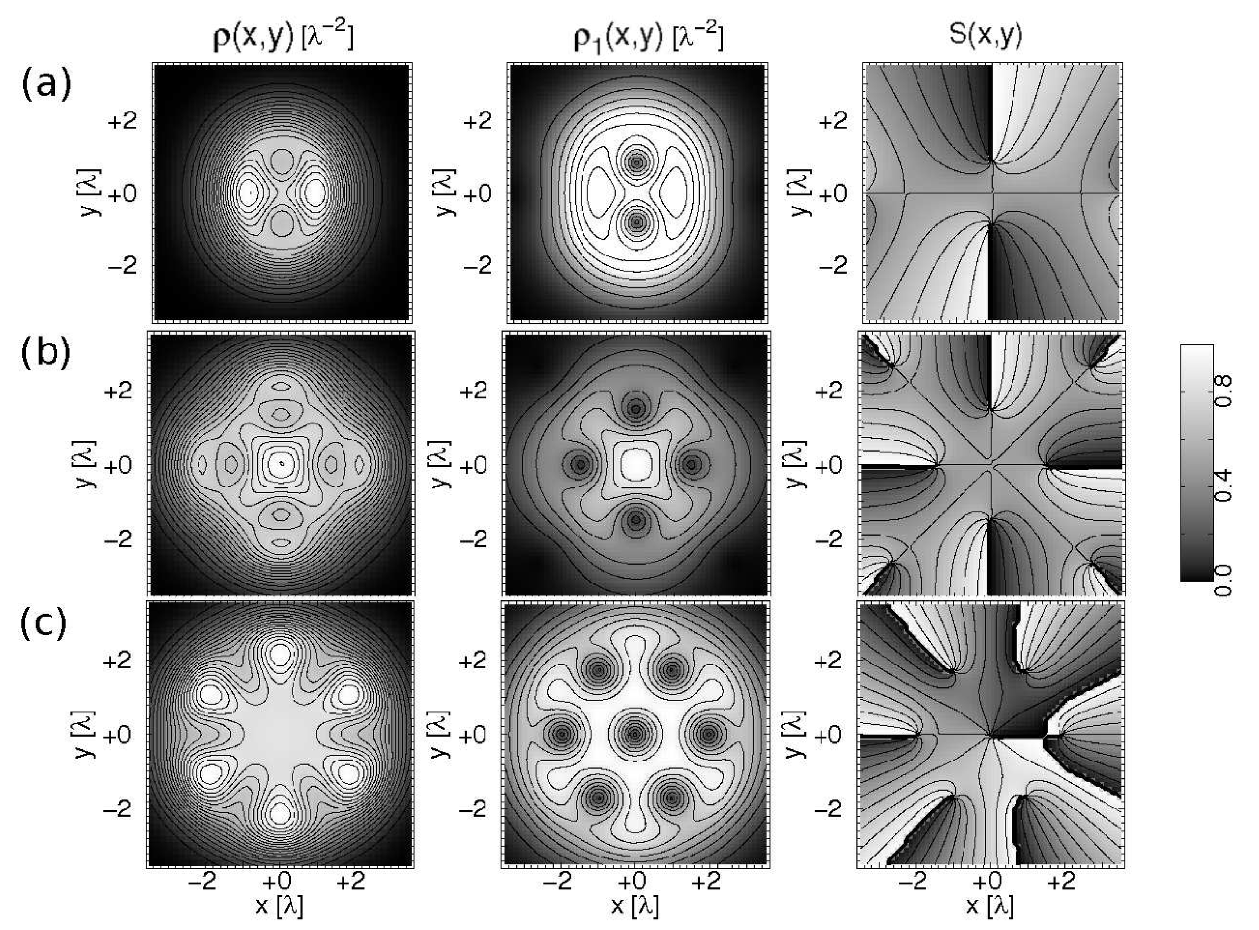


Fig. 1: For $N = 6$ the first two pictures on each row show the density contour plot of the GS ($\rho(x, y)$) and the $\psi_1$ function ($\rho_1(x, y)$) respectively. The third picture shows tha map of the phase $S(\vec{r})$ (see text). (a) shows a two vortex structure at $\Omega = 0.941$ (where degeneracy between $L = 10$ and 12 takes place). (b) shows a four vortex structure, $\Omega = 0.979$ (degeneracy between $L = 20, 22$ and 24). (c) shows a six-fold structure, $\Omega = 0.983$ (degeneracy betwee $L = 24, 26, 28$ and 30). In all cases $\omega_\perp = g = 1$ in units of $\lambda$ and $u = \hbar\omega_\perp$.

### Interferences

We represent the two independent condensates which we call $a$ and $b$ by their macroscopic occupied function $\psi_a$ and $\psi_b$ respectively.

- At time $t = 0$ s the condensates are separated by a distance $d$ and the traps are switched off.
- The time evolution of the system is obtained (once the transformation to the laboratory frame of reference is performed, multiplying the functions by $exp(-i\Omega t \hat{L}_z)$) in three steps:
  1. The Fourier transform of the total order parameter (the sum of the two contributions) is performed.
  2. The time evolution of the Fourier components by multiplying them by exponentials of the type $exp(i\hbar k^2 t/2m)$ is realised; this step is done under the assumption that during the time-of-flight the interactions are irrelevant.
  3. Inverse Fourier transformation is performed.

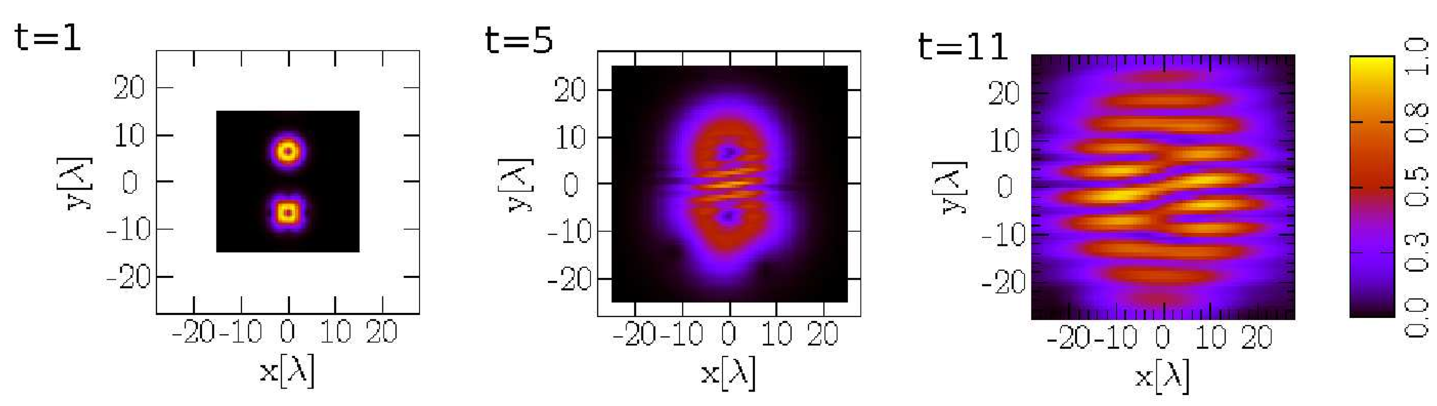


Fig. 2: Time evolution of the interference pattern during the overlap of two released condensates initially separated by a distance $d = 15\lambda$. Initially each condensate contains $N = 6$ atoms and their GS are characterized by $L = 6$ at $\Omega = 0.019$ and by a mixture of $L = 6, 8$ and 10 at $\Omega = 0.0847$ respectively (all quantities are in units of $\lambda$ and $u$).

## Conclusions

- The use of the eigenfunctions of the OBDM operator provides a useful and precise tool to analyze the exact GS obtained from exact diagonalization and specially the vortex states. These eigenfunctions:
  - Localize and quantize the vortices.
  - Reproduce the time evolution of the interference pattern of two overlapping condensates.
- This last result imply an alternative interpretation about the interference pattern formation. In our case, the real initial states are Fock states and no interaction has been included. We observe an interference pattern due to the definition of the phase induced by the fluctuations of the number of condensed atoms.

## References

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